

ON THE THEORY OF HYPERSONIC FLOW OVER BLUNT-NOSED SLENDER BODIES

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Summary—This paper is concerned with the two-dimensional and axially symmetric hypersonic flows over blunt-nosed slender bodies. On the basis of investigation of the entropy layer adjacent to the body surface it is shown, that for practical application of the small-disturbance theory of hypersonic flow more precise knowledge of the entropy layer thickness is required.

The method is given to introduce a correction to that effect and to plot a shape of the body contour to which pressure distribution obtained on the basis of the small-disturbance theory should be referred.*

1. ACCORDING to the small-disturbance theory of hypersonic flow, the problem of flow around a two-dimensional or axisymmetric body of low thickness ratio is equivalent to the problem of one-dimensional unsteady gas flow due to the motion of a two-dimensional or cylindrical piston ⁽¹⁾.

Within this analogy, the class of the similar flows with strong shock waves propagating conforming to a power law⁽²⁾ corresponds to the class of steady flows with shock waves of power-law shape

$$y = Cx^n \quad (1.1)$$

with the Mach Number of the undisturbed flow $M_\infty \rightarrow \infty$. The values of the exponent n , varying within the range

$$\frac{2}{3+\nu} < n < s \quad (1.2)$$

($\nu = 0$ and 1 for two-dimensional and axisymmetric flow respectively) correspond to the flows around a power law convex body⁽³⁾:

$$y = cx^n \quad (1.3)$$

The case

$$n = \frac{2}{3+\nu} \quad (1.4)$$

is a singular one and corresponds to the blast wave problem⁽²⁾.

* Some of this paper's results have been published⁽¹¹⁾

As in this case the relation $c/C = 0$, treatment of this case with respect to the problem of steady flow is based on the assumption, that the finite drag force is applied to the leading edge of the body of a vanishing thickness. In other words, the analogy is drawn between a blast wave phenomenon and the effect of the blunting of a slender body at a large distance from the leading edge^(4, 5, 6).

As it is known⁽⁶⁾, for the general case the principle of equivalence of the flow around a blunt-nosed slender airfoil or body of revolution of arbitrary shape leads to the problem of one-dimensional unsteady gas

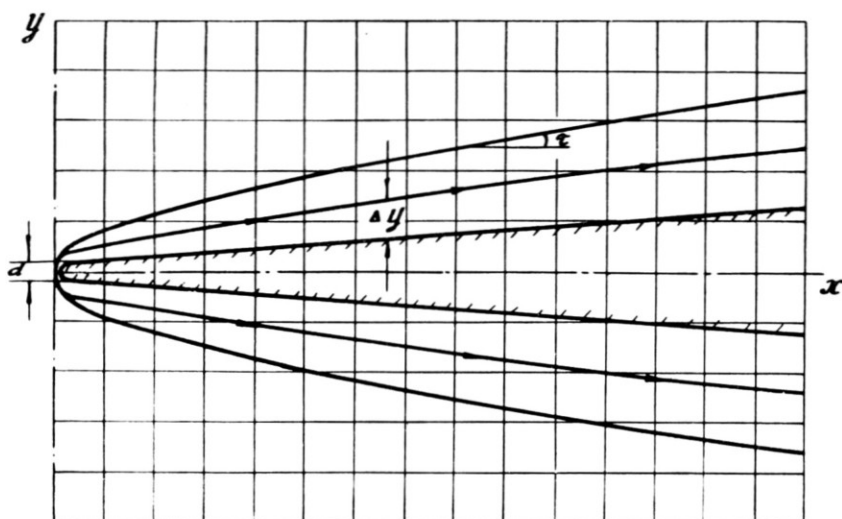


FIG. 1.

flow due to the action of a strong explosion on a plane or a straight line with simultaneous influence of a two-dimensional or cylindrical piston which begins expanding at the same moment of time.

The energy of explosion E per unit of area or length of charge in this case is equalized to the drag of a blunting which is assumed to be known from other considerations.

2. The small-disturbance theory of hypersonic flow cannot be applied in the vicinity of the tip of the shock wave enveloping a blunt-nosed body, as the velocity perturbations here are finite. The entropy values on the streamlines crossing the surface of the shock wave in this region are highly overestimated, and on the surface of the body ($\psi = 0$) the entropy appears to be infinitely large, so that the density here is reduced to zero.

As a result, the small-disturbance theory is inapplicable in the whole flow region of large entropy adjacent to the body surface (Fig. 1).

Consider the flow in this region in detail.

3. The equations of two-dimensional or axisymmetric gas flow after a von Mises transformation from independent variables x, y to independent variables x, ψ

(ψ —stream function) take the form

$$\frac{\partial p}{\partial \psi} = -\frac{s}{y^v} \frac{\partial v}{\partial x} \quad (3.1)$$

$$\frac{\partial}{\partial x} \left(\frac{p}{\rho^{\gamma}} \right) = 0 \quad (3.2)$$

$$\frac{\partial y}{\partial \psi} = \frac{s}{\rho u y^v} \quad (3.3)$$

$$\frac{\partial y}{\partial x} = \frac{v}{u} \quad (3.4)$$

$$u^2 + v^2 + \frac{2\gamma}{\gamma - s} \frac{p}{\rho} = U_{\infty}^2 \quad (3.5)$$

Where u, v —components of velocity vector, p —pressure, ρ —density, γ —gas specific heats ratio. The subscript ∞ here and below refers to the undisturbed flow conditions, the static pressure of which is neglected. The boundary conditions on the surface of the shock wave (1.1) are of the form:

$$\psi = \frac{s}{s+v} \rho_{\infty} U_{\infty} Y^{1+v}(x) \quad (3.6)$$

$$u = U_{\infty} \left[s - \frac{2}{\gamma+s} \frac{Y'^2(x)}{s+Y'^2(x)} \right] \quad (3.7)$$

$$v = \frac{2}{\gamma+s} U_{\infty} \frac{Y'(x)}{s+Y'^2(x)} \quad (3.8)$$

$$p = \frac{2}{\gamma+s} \rho_{\infty} U_{\infty}^2 \frac{Y'^2(x)}{s+Y'^2(x)} \quad (3.9)$$

$$\rho = \frac{\gamma+s}{\gamma-s} \rho_{\infty} \quad (3.10)$$

$$y = Y(x) \quad (3.11)$$

The corresponding relations of the small-disturbance theory are obtained, if in the equation (3.3), (3.4) and the boundary conditions (3.8), (3.9) we assume

$$u \cong U_{\infty} \quad \text{and} \quad s+Y'^2(x) \cong s$$

The equations (3.5), (3.7) are then omitted and the remaining set is equivalent to the set of equations and boundary conditions of the one-dimensional unsteady flow in terms of Lagrange variables.

4. First let us use the explicit equations of Section 3 to evaluate the pressure difference across the entropy layer near the body surface. Consider this layer as a flow region formed by the streamlines crossing the surface of the shock wave near its tip, where the angles of inclination of this surface to the oncoming stream are not small:

$$y'(x) \geq s \quad (4.1)$$

If we denote the thickness or diameter of the body nose by d , then, the increment of the stream function across the entropy layer is obtained by means of equation (3.6):

$$\Delta\psi \sim \rho_\infty U_\infty d^{1+\nu} \quad (4.2)$$

For the particular case of power-law shock shape

$$Y(x) = Cx^n \quad (4.3)$$

on the basis of the expressions (4.1) and (3.6) we have

$$\Delta\psi \sim \rho_\infty U_\infty C^{\frac{1+\nu}{1-n}} \quad (4.4)$$

Everywhere outside the region of the body nose the angle of the shock wave inclination τ may be considered small and the following order estimates may be written for the variables ν , p and y :

$$\nu \sim U_\infty \tau, \quad p \sim \rho_\infty U_\infty^2 \tau^2, \quad y \sim \tau x \quad (4.5)$$

and for the power-law shock waves

$$\tau \sim \frac{C}{x^{1-n}} \quad (4.6)$$

Substituting these values in the equation (3.1), we determine that the relative pressure differences across the entropy layer is of the order:

$$\frac{\Delta p}{p} \sim \left(\frac{d}{x\tau} \right)^{1+\nu} \sim \tau^{\frac{n(1+\nu)}{s-n}} \quad (4.7)$$

Hence, for the general case when

$$x \geq \frac{d}{\tau^{\frac{3+\nu}{1+\nu}}} \quad (4.8)$$

and for the particular case (4.3) when $n \geq \frac{2}{3+\nu}$

$$\frac{\Delta p}{p} \leq \tau^2 \quad (4.9)$$

Thus within the accuracy of the small-disturbance theory (leading, as we may easily see, to the same result) the pressure difference across

the entropy layer at sufficiently large distances from the leading edge may be neglected. Therefore, the relation $p(x, \psi)$, obtained on the basis of this theory (but not $p(x, y)$, as we shall see later), at the indicated distances from the leading edge is uniformly valid within the whole flow field, the entropy layer included.

5. To evaluate the relative thickness of the entropy layer, we shall take the equations (3.2) and (3.3). From the condition of constancy of entropy along the streamlines (3.2) and boundary conditions (3.9), (3.10) we can determine that along the whole entropy layer

$$\frac{p}{\rho^\gamma} \sim \frac{\rho_\infty U_\infty^2}{(\rho_\infty K)^\gamma} \quad (5.1)$$

where $K = \frac{\gamma+1}{\gamma-1}$. Using the estimated pressure value (4.5), we find that the density is of the order:

$$\rho \sim \rho_\infty K \tau^{2/\gamma} \quad (5.2)$$

Then, assuming $u \sim v_\infty$ and using the equation (3.3) we find that the relative thickness of the entropy layer is of the order:

$$\frac{\Delta y}{y} \sim \frac{\Delta p}{p} \cdot \frac{1}{K \cdot \tau^{2/\gamma}} \quad (5.3)$$

It is negligibly small only at the distances x of the order of $d/\tau^{\frac{3+v+2/\gamma}{1+v}}$, by far exceeding the value (4.8), and for the particular case (4.3) only with

$$n \geq \frac{2+2/\gamma}{3+v+2/\gamma} *$$

At the same time, when solving the problem on the basis of the small-disturbance theory, the region of the entropy effect is many times larger, due to which the body contour, corresponding to the shock wave of the given shape, is determined with a larger error. In fact, when determining the relative thickness of the entropy layer, we use in this case instead of (3.9) the approximate equation for p on the shock wave surface of the form

$$p = \frac{2}{\gamma+1} \rho_\infty V_\infty^2 y'^2(x) \quad (5.4)$$

This leads, for instance in case of the power-law shock wave in the vicinity of the nose, to the following value of the density:

$$\rho \sim \rho_\infty K \tau^{2/\gamma} C^{-2/n\gamma} \left(\frac{\rho_\infty V_\infty}{\psi} \right)^{-\frac{2-2n}{n(1+v)}} \quad (5.5)$$

* The order estimates analogous to (4.7) and (5.3) for the case of two-dimensional flows with power-law shock waves were obtained in the paper⁽¹²⁾.

As a result, integrating the equation (3.3) across the entropy layer we obtain:

$$\frac{\Delta y}{y} \sim \frac{N}{K} \cdot \tau^{\frac{n(+v)}{1-n} - \frac{2}{\gamma}} \sim \frac{\Delta p}{p} \cdot \frac{N}{K\tau^{2/\gamma}} \quad (5.6)$$

where

$$N = \frac{1}{1 - \frac{2}{\gamma} \cdot \frac{1-n}{n(1+v)}} \quad (5.7)$$

With decrease in γ and n the factor N may increase to large values (Fig. 2), and the relative entropy layer thickness, determined on the basis of the small-disturbance theory, can exceed its actual thickness many times.

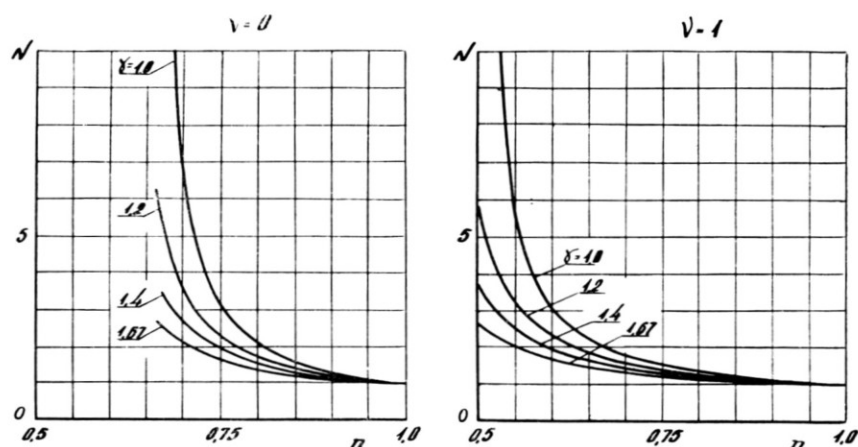


FIG. 2.

6. Thus, utilizing the equivalence principle of the small-disturbance theory of hypersonic flow in the problems of the steady flow over blunt-nosed slender bodies requires more precise knowledge of the entropy layer thickness, which (for the given shock shape) must lead to the corresponding correction of the body contour. According to Section 4 the pressure distribution over the body surface, obtained on the basis of this theory, must be referred to the new contour of the body. Practically, it is convenient to determine the shape of the body corresponding to the given shock shape (3.11) by integrating the equation (3.3) across the whole flow field for a number of fixed Xs .

The function $p(x, \psi)$ here is known from solution of the corresponding problem on the basis of the small-disturbance theory, and the relation

$$\frac{p}{\rho_\gamma} = q(\psi) \quad (6.1)$$

is known from the exact boundary conditions (3.6), (3.9), (3.10). Thus we determine the density $\varrho(x, \psi)$.

Now we must determine more precisely the component of the velocity vector u involved in (3.3) which was assumed above to be of the order V_∞ .

As everywhere, with the exception of the shock wave tip region,

$$\frac{v}{u} \sim \tau$$

then on the basis of energy equation (3.5) we have

$$u \cong \sqrt{U_\infty^2 - \frac{2\gamma}{\gamma-1} \frac{p}{\varrho}} \quad (6.2)$$

with a relative error of the order of τ^2 .

Substituting in this equation the order estimates for p and ϱ obtained earlier (4.5), (5.2), we determine, that

$$\left(\frac{u}{U_\infty}\right)^2 - 1 \sim \tau^{2-2/\gamma} \quad (6.3)$$

from which it follows that the equivalence principle is invalid in the entropy layer and that it is necessary to utilize the formula (6.2) to determine u (to the required degree of accuracy). This equation closes the set of relations required for calculation.

7. To determine the drag of the body, it is convenient to use the integrated form of the momentum equation, which may be obtained on the basis of (3.1) to (3.4) and in the limiting case of the flow with $M_\infty \rightarrow \infty$ takes the form:

$$\oint \left(\frac{p}{\varrho u} + u - V_\infty \right) d\psi + py \frac{v}{u} dx = 0 \quad (7.1)$$

Taking the zero streamline, the shock wave line and the straight line $x = l = \text{const}$, as a contour of integration, we obtain (using the boundary conditions on the shock wave surface and the body surface) the following expression for the drag x :

$$x = (2\pi)^{\nu} \int_{\frac{1}{1+\nu} \rho_\infty V_\infty Y^{1+\nu(l)}}^0 \left(\frac{p}{\varrho u} + u - V_\infty \right) d\psi \quad (7.2)$$

The functions $p(x, \psi)$, $\varrho(x, \psi)$, $u(x, \psi)$ involved in this formula have already been determined.

It should be noted, that though the shape of the body blunting cannot be determined on the basis of the method under discussion, the drag of this portion of the body is determined by the formula (7.2) to the same degree of accuracy, as the drag of the remainder of the body.

8. Let us now consider more thoroughly the solution of the problem for a particular case of the flow with power-law shock waves (4.3). The corresponding unsteady flow of gas in this case is similar. Usually the results of integrating the equations of the similar flows of gas are expressed in the form^(2, 3):

$$v = v_s(x)f(\lambda), \quad \varrho = \varrho_s(x)g(\lambda), \quad p = p_s(x)h(\lambda) \quad (8.1)$$

where

$$\lambda = \frac{y}{y_s}$$

and the subscript S refers to the shock wave conditions.

The stream function for similar flow required for calculation, satisfying the differential equation

$$V_\infty \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y} = 0 \quad (8.3)$$

may be readily determined as

$$\psi = \psi_s(x) \cdot \eta(\lambda) \quad (8.4)$$

where

$$\eta(\lambda) = \exp \left\{ -(1+v) \int_1^\lambda \frac{d\lambda}{\frac{2}{\gamma+1} f(\lambda) - \lambda} \right\} \quad (8.5)$$

This, together with (8.1), defines the relation $p(x, \eta)$ which we can now write, using (5.4) and (4.3), in the form:

$$p = \frac{2}{\gamma+1} \varrho_\infty V_\infty^2 n^2 C^2 \frac{H(\eta)}{x^{2(1-n)}} \quad (8.6)$$

The entropy distribution function is determined from (3.6), (3.8), (3.9) and, taking into account (8.4), may be expressed:

$$\frac{p}{\varrho^\gamma} = \left(\frac{\gamma-1}{\gamma+1} \frac{1}{\varrho_\infty} \right)^\gamma \frac{2}{\gamma+1} \varrho_\infty V_\infty^2 \frac{n^2 C^2}{x^{2(1-n)} \eta^{\frac{2(1-n)}{\gamma+1}} + n^2 C^2} \quad (8.7)$$

Eliminating p and ϱ from the last two equations, and then using the equation (6.2) we define U . Then

$$\frac{\varrho_\infty}{\varrho} = \frac{\gamma-1}{\gamma+1} G(x, \eta) \quad (8.8)$$

$$\frac{u}{V_\infty} = \left[s - \frac{4\gamma}{(\gamma+1)^2} n^2 C^2 \frac{H(\eta) G(x, \eta)}{x^{2(1-n)}} \right]^{1/2} \quad (8.9)$$

where

$$G(x, \eta) = \left[\frac{x^{2(1-n)}}{x^{2(1-n)} \gamma^2 (1-\nu) + n^2 C^2} \cdot \frac{1}{H(\eta)} \right]^{1/\nu} \quad (8.10)$$

The equation (3.1) by changing the independent variable ψ for η is reduced to take the form

$$y \frac{\nu \partial y}{\partial \eta} = \frac{\psi_0(x)}{\rho u} \quad (8.11)$$

Substituting the determined function $\rho(x, \eta)$, $u(x, \eta)$ in this equation and integrating with fixed Xs , we obtain the equation which determines the shape and location of the streamlines of the flow under consideration:

$$y(x, \eta) = Cx^n \left\{ s + \frac{\gamma-s}{\gamma+1} \int_s^\eta \frac{G(x, \eta) d\eta}{\sqrt{s - \frac{4\gamma}{(\gamma+s)^2} n^2 C^2 \frac{H(\eta) G(x, \eta)}{x^{2(1-n)}}}} \right\}^{\frac{1}{s+\nu}} \quad (8.12)$$

The body contour can be determined by this equation, if the upper limit of the integral is assumed to be zero. To determine the drag of the body, let us use the equation (7.2). Substituting the values p , ρ and u determined from (8.6), (8.8) and (8.9) in this formula we shall obtain:

$$X = \frac{(2\pi)^\nu}{s+\nu} \rho_\infty V_\infty^2 (Cx^n)^{1+\nu} \left\{ \frac{2(\gamma-1)}{(\gamma+s)^2} \frac{n^2 C^2}{x^{2(1-n)}} \int_0^s \frac{H(\eta) G(x, \eta) d\eta}{\sqrt{s - \frac{4\gamma}{(\gamma+s)^2} n^2 C^2 \frac{H(\eta) G(x, \eta)}{x^{2(1-n)}}}} + \int_0^1 \left[\sqrt{s - \frac{4\gamma}{(\gamma+1)^2} n^2 C^2 \frac{H(\eta) G(x, \eta)}{x^{2(1-n)}}} - 1 \right] d\eta \right\} \quad (8.13)$$

9. Consider some results of numerical calculations for the axisymmetric case ($\nu = 1$) with the shock waves varying according to the power law

$$y = Cx^n \quad (9.1)$$

The calculations were performed for the case with $\gamma = 1.4$. The functions (8.1) were taken from the tables presented in references 2 and 7.

In Figs. 3 and 4 is given a comparison between the contours of the bodies, obtained on the basis of the present method, and the corresponding results of the small-disturbance theory (the latter are plotted everywhere by dotted lines). As we may see, the differences between them are quite negligible already at $n = 0.65$, but at $n = 0.5$ the differences are very large. The relative thicknesses of the bodies

$$\delta(x) = \frac{y(x, \alpha)}{y(x, 1)} \quad (9.2)$$

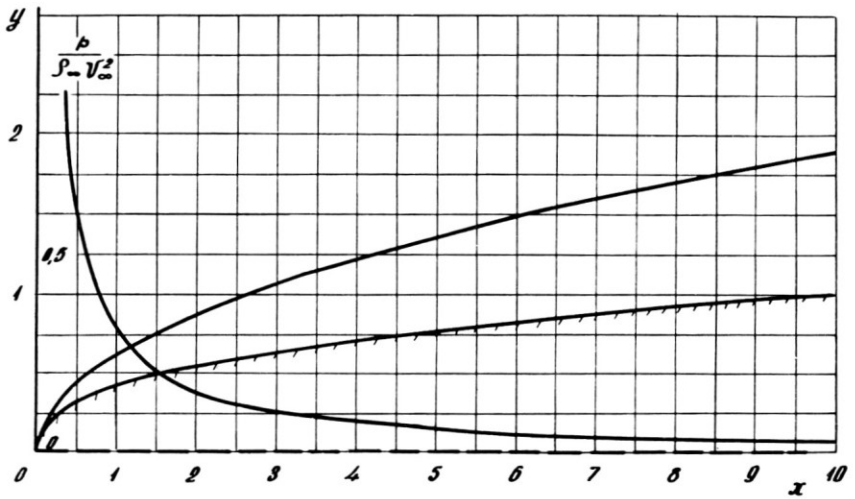
$n = 0,50$ 

FIG. 3.

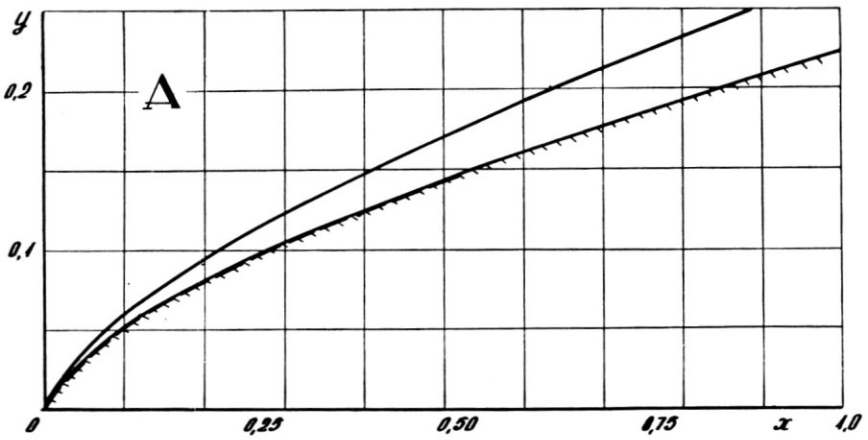
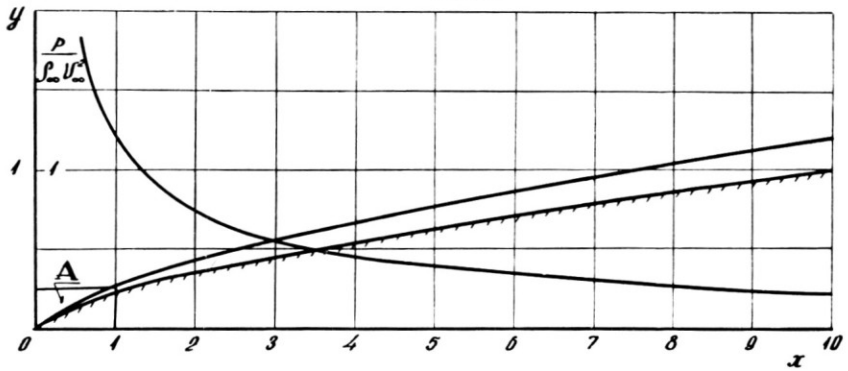
 $n = 0,65$ 

FIG. 4.

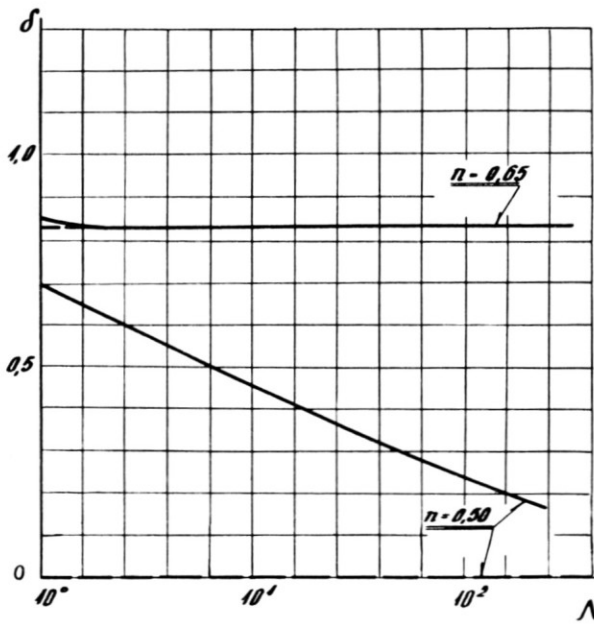


FIG. 5.

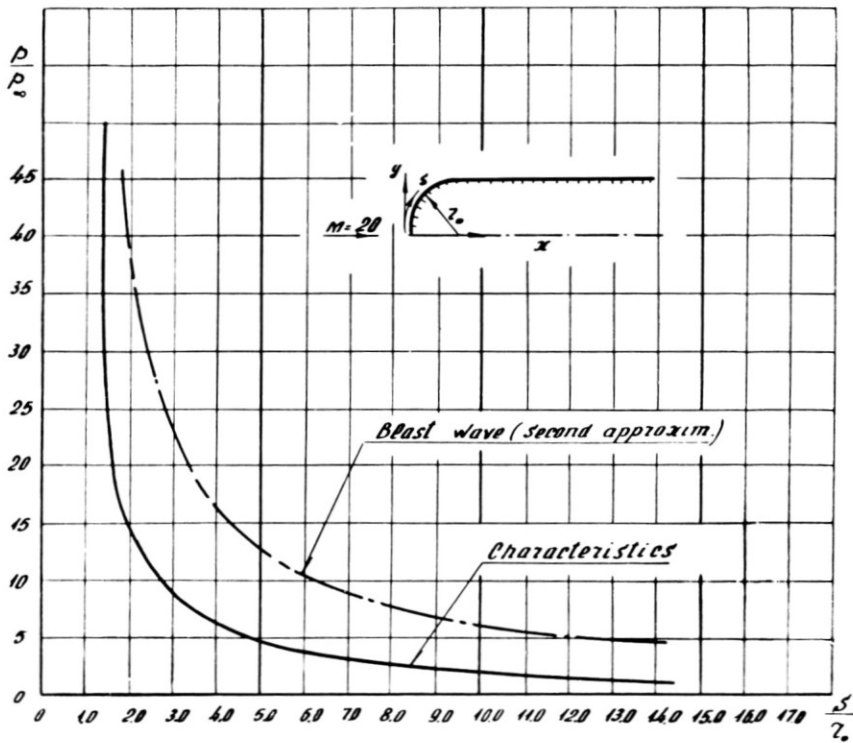


FIG. 6.

calculated on the basis of (8.11) are presented in Fig. 5 as functions of their fineness ratio

$$A = \frac{x}{2y(x, a)} \quad (9.3)$$

As we may see, the relative thickness of the body corresponding to the case $n = 0.5$ is far from being negligible even in the region of very large values of $A \sim 10^2$.

Thus, the pressure distribution prescribed by the blast wave solution to the blunt-nosed cylinder is actually realized on a body of a considerably larger thickness ratio. Probably, this may explain a considerable numerical difference between the pressure distribution on the surface of the cylinder with hemispherical nose obtained in this way (taking into account finite counterpressure) and the results of numerical calculation⁽⁸⁾ for $M_\infty = 20$ (Fig. 6).

Presented in Fig. 7 is a comparison between the velocity and density profiles in the section $A = 1.76$, calculated by the formulas (8.8), (8.12) for $n = 0.5$ and the corresponding results of the constant energy solution⁽²⁾.

10. The solution of the problem for the general case of the flow with the shock formed on a blunt-nosed slender body of arbitrary shape present considerable difficulties in view of the fact, that the corresponding solution on the basis of the small-disturbance theory here is not similar.

To obtain approximate solution of the problem in this case, we may use the well-known method of successive approximation^(7, 9), based on the expansion of the functions, to be determined, in power series of the form:

$$\begin{aligned} v &= V_\infty [Y'(x) + \varepsilon v_1 + \dots] \\ p &= \varrho_\infty V_\infty^2 (p_0 + \varepsilon p_1 + \dots) \\ \varrho &= \frac{\varrho_\infty}{\varepsilon} (\varrho_0 + \varepsilon \varrho_1 + \dots) \\ y &= Y(x) [s + \varepsilon y_1 + \dots] \end{aligned} \quad (10.1)$$

where $\varepsilon = \frac{\gamma-1}{\gamma+1}$. The main drawback of this method is its inapplicability to the calculation of flows with the regions of the small density, where the initial assumption, that the flow is of a thin shock layer type, does not hold. However, the only purpose of using this method here is determination of the function $p(x, \psi)$, with respect to which this method possesses a high degree of accuracy. The last is due to the fact that a pressure difference in a low density region can not be considerable. Let us consider this in greater detail for the case of flows with the shock waves

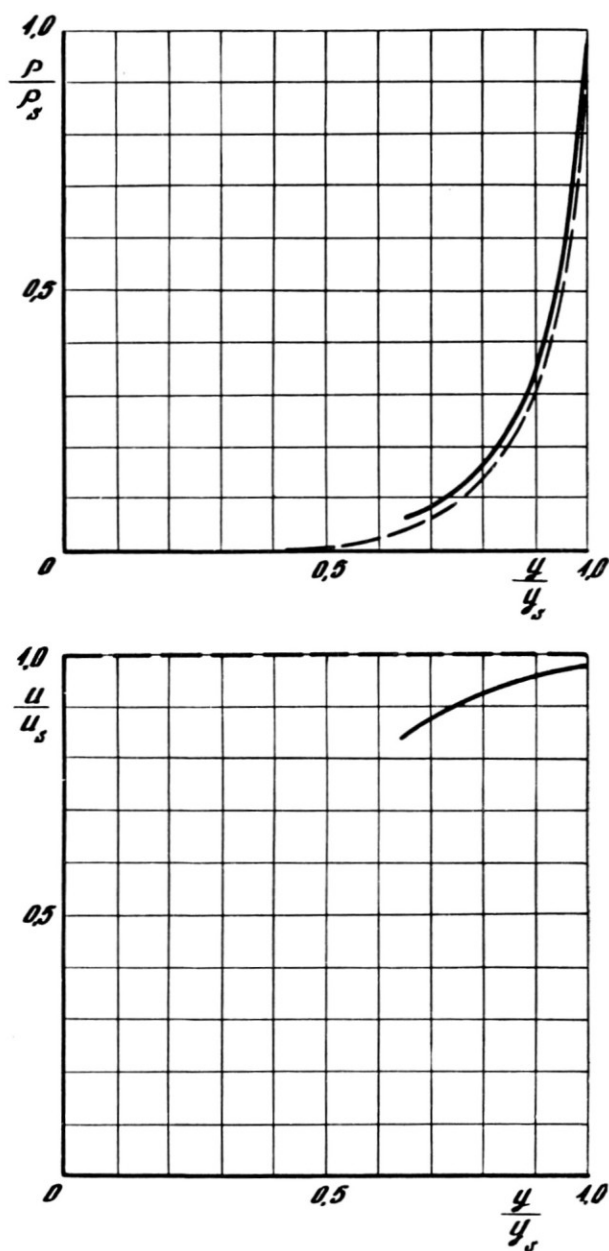


FIG. 7.

of power law shape. Defining the region of inapplicability of the method of successive approximation as the part of flow field where

$$\varrho \leq \varrho_{\infty} \quad (10.2)$$

and using the equation (9.2), boundary conditions (3.6), (3.10), (5.4) and pressure estimation (4.5) we may obtain for variation of the streamline function in the region under consideration the expression

$$\Delta\psi \leq \frac{\varrho_{\infty} V_{\infty} C^{1+\nu} \chi^{n(1+\nu)}}{K^{\frac{2(1-n)}{1+\nu}}} \quad (10.3)$$

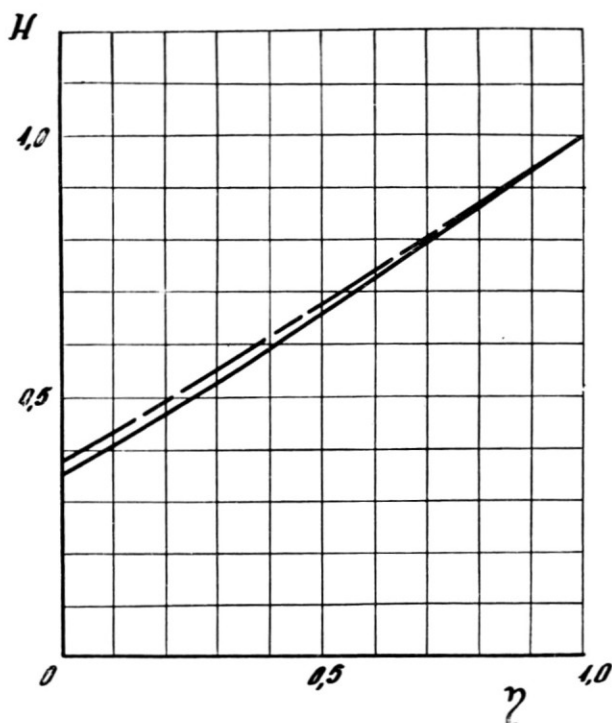


FIG. 8.

Then the equation (3.1) evaluates the pressure difference across the small density layer as follows:

$$\frac{\Delta p}{p} \leq \frac{1}{K^{\nu}} \left(n \geq \frac{2}{3+\nu} \right) \quad (10.4)$$

from which it follows that this ratio is small at sufficiently small $\varepsilon = \frac{1}{K}$.

Presented in Fig. 8, as an example, is the relation $H(\eta)$ to be found (8.6) for the case corresponding to the cylindrical blast wave solution $\nu = 1$,

$n = \frac{1}{2}$, when the method of successive approximation is the most doubtful.

The appropriate formula of the second-order approximation is:

$$H(\eta) = \frac{1+\eta}{2} + \varepsilon \left[2 \ln \frac{1+\eta}{2} + \frac{(1+\eta)^2}{2(1+\eta)} \right] \quad (10.5)$$

and, as we see, its agreement with the exact solution is quite satisfactory (at $\gamma = 1.4$; $\varepsilon = 1/6$).

11. All the results obtained in this paper referred to the hypersonic flows of the perfect gas with the constant specific heats for the limiting case of $M_\infty \rightarrow \infty$.

The extension of the results for the cases of the thermodynamically equilibrium real-gas flow with finite values of M_∞ presents no principal difficulties.

A substantial simplification may be achieved in this case by dividing the problem in two independent parts.

Taking into account the value M_∞ being finite (the finite counterpressure) is evidently significant only when determining the pressure distribution function $p(x, \psi)$, i.e. it may be carried out by the usual methods^(2, 9) when solving the corresponding problem on the basis of the small-disturbance theory.

On the other hand, taking into consideration the real-gas properties at high temperature is of importance only in the entropy layer of the flow, formed by the streamlines crossing the shock wave in the region of its highest intensity. This allows, when solving the second part of the problem (determining the relations $\varrho(x, \psi)$, $u(x, \psi)$ and integrating the equation (3.1)), to a sufficient degree of accuracy, to use the approximate shock wave relations without taking the finite counterpressure into account.

These relations, as well as the energy equation and the adiabatic condition, should be changed for relations which take into account the real dependence of the specific enthalpy and the specific entropy of the gas upon the pressure and density.

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